Derivatives of inverses

Recall: f and g are inverse functions iff

$f(g(x)) = x$ for every x in domain of g

$g(f(x)) = x$ for every x in the domain of f

So [differentiating implicitly]

$$f(g(x)) = x$$

chain rule

$$f'(g(x))\frac{d}{dx}(g(x)) = 1$$

$$f'(g(x))\,g'(x) = 1$$

$$g'(x) = \frac{1}{f'(g(x))}$$

You can use this relationship $\left[g'(x) = \frac{1}{f'(g(x))} \text{ when } g = f^{-1} \right]$ to check what you already know.

For convenience, let $\exp(x) = e^x$
let $\text{sqrt}(x) = \sqrt{x}$

for example:

$$\exp(\ln(x)) = x$$

differentiating implicitly

$$\exp(\ln(x)) \cdot \frac{d}{dx}(\ln(x)) = 1$$

$$\frac{d}{dx}(\ln(x)) = \frac{1}{\exp(\ln(x))} = \frac{1}{x}$$

$$\ln(\exp(x)) = x$$

$$\frac{1}{\exp(x)} \frac{d}{dx}(\exp(x)) = 1$$

$$\frac{d}{dx}(\exp(x)) = \exp(x) = e^x$$

$\frac{d}{dx}(e^x) = e^x$
b/c
x is exponent
not base.
power rule "works" ONLY when exponent is constant

Another example, for certain domain restrictions

$(\text{sqrt}(x))^2 = x$

differentiate implicitly

$2(\text{sqrt}(x)) \cdot \frac{d}{dx}(\text{sqrt}(x)) = 1$

$\frac{d}{dx}(\text{sqrt}(x)) = \frac{1}{2\,\text{sqrt}(x)}$

$\frac{d}{dx}(x^{\frac{1}{2}}) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

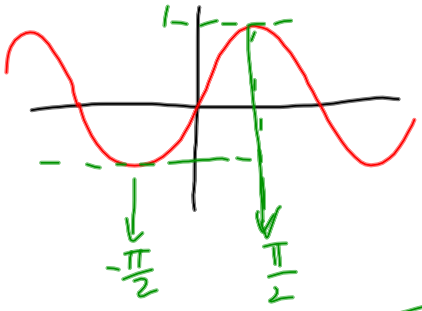
$\text{sqrt}(x^2) = x$

$\frac{1}{2\,\text{sqrt}(x^2)} \frac{d}{dx}(x^2) = 1$

$\frac{d}{dx}(x^2) = 2\,\text{sqrt}(x^2)$

$\frac{d}{dx}(x^2) = 2x$

4.4 derivatives of inverse trig fns



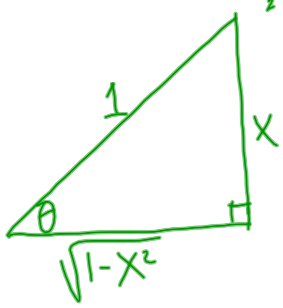
$\sin^{-1}(x) = \arcsin(x)$

= "the angle [θ] whose sin is x, and which is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$"

$\sin(\sin^{-1}(x)) = x$

differentiate implicitly

$\cos(\sin^{-1}(x)) \cdot \frac{d}{dx}(\sin^{-1}(x)) = 1$



$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\cos(\sin^{-1}(x))} = \frac{1}{\sqrt{1-x^2}}, \; x \in [-1,1]$$

$x \in [-1,1]$

x "is in" "is an element of" "the closed interval between -1 and 1" → includes endpoints

$\sin(x)$: domain: $(-\infty, \infty)$

restricted domain: $(-\frac{\pi}{2}, \frac{\pi}{2})$

range: $[-1,1]$

$\sin^{-1}(x)$: domain: $[-1,1]$

range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

So: the derivatives of trig functions

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$

$$* \quad \frac{d}{dx}\left(\sec^{-1}(x)\right) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\cot^{-1}(x)\right) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left(\csc^{-1}(x)\right) = \frac{-1}{|x|\sqrt{x^2-1}}$$

Oct 31-7:32 AM